

# Model-independent dark energy test with $\sigma_8$ using results from the Wilkinson Microwave Anisotropy Probe

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By combining the recent WMAP measurements of the cosmic microwave background anisotropies and the results of the recent luminosity distance measurements to type-Ia supernovae, we find that the normalization of the matter power spectrum on cluster scales,  $\sigma_8$ , can be used to discriminate between dynamical models of dark energy (quintessence models) and a conventional cosmological constant model ( $\Lambda$ CDM).

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## I. INTRODUCTION

The WMAP satellite measurements of the Cosmic Microwave Background anisotropies [1] have provided accurate determinations of many of the fundamental cosmological parameters. When combined with other data sets such as the luminosity distance to type-Ia supernovae or large scale structure (LSS) data [3, 4, 5, 6], they reinforce the need for an exotic form of dark energy, which is characterized by a negative pressure and is responsible for the observed accelerated expansion of the universe. There are two main scenarios used to explain the nature of the dark energy, a time independent cosmological constant  $\Lambda$  and quintessence, which involves an evolving scalar field  $Q$  [7, 8, 9]. Previous tests of quintessence with pre-WMAP CMB data [10, 11, 12], have led to constraints on the value of the dark energy equation of state parameter,  $w_Q \lesssim -0.7$  with the cosmological constant value,  $w_\Lambda = -1$  being the best fit. Nevertheless a dynamical form of dark energy is not excluded. Specifically the detection of a time variation in this parameter would be of immense importance as it would rule out a simple cosmological constant scenario. When parameterising quintessence models we do not want to assume simply a constant equation of state  $w_Q$  since this introduces a systematic bias in the analysis of cosmological distance measurements [13], with the effect of favouring larger negative values of  $w_Q$  if the dark energy is time dependent. For instance it is possible that claims for a ‘phantom’ component, where  $w_Q < -1$  [11, 14] are entirely caused by this effect. Moreover assuming  $w_Q$  constant underestimates the contribution of the dark energy perturbations (which are a specific feature of scalar field models) on the evolution of the gravitational potentials and consequently the effect on the CMB power spectrum [15]. In this paper we deliberately do not assume  $w_Q$  to be constant, rather we focus on the relation between a dynamical dark energy component and the normalisation of the dark matter power spectrum on cluster scales,  $\sigma_8$ . We also discuss the age of the universe,  $t_0$ , and show how the new data sets undermine its use for distinguishing be-

tween different dark energy models.

## II. METHOD AND DATA

In this analysis, rather than considering a specific scalar field model, we allow for a time dependence of the equation of state parameter  $w_Q$ . Several formulae have been proposed in the literature [16, 17] all with limited applicability. In [18] a form for  $w_Q(z)$  was suggested which is valid at all redshifts and parameterises the equation of state in terms of five parameters, which specify the value of the equation of state parameter today  $w_Q^0$ , and during the matter/radiation eras  $w_Q^m/w_Q^r$ ; the scale factor  $a_c^m$  where the equation of state changes from  $w_Q^m$  to  $w_Q^0$  and the width of the transition  $\Delta$ . Since Big-Bang Nucleosynthesis bounds limit the amount of dark energy to be negligible during the radiation dominated era, without loss of generality we can further reduce our parameter space by setting  $w_Q^r = w_Q^m$  in Eq. (4) of Ref. [18]. The parameters given by the vector  $\overline{W}_Q = (w_Q^0, w_Q^m, a_c^m, \Delta)$  can account for most of the dark energy models proposed in the literature. For instance quintessence models characterized by a slowly varying equation of state, such as supergravity inspired models [19], correspond to a region of our parameter space for which  $0 < a_c^m/\Delta < 1$ , while rapidly varying models, such as the two exponential potential case [20], correspond to  $a_c^m/\Delta > 1$ . Models with a simple constant equation of state are given by  $w_Q^0 = w_Q^m$ . The cosmological constant case is also included and corresponds to the following cases:  $w_Q^0 = w_Q^m = -1$  or  $w_Q^0 = -1$  and  $a_c^m \lesssim 0.1$  with  $a_c^m/\Delta > 1$ . Assuming a flat geometry we perform a likelihood analysis of the WMAP data to constrain dark energy models specified by the vector  $\overline{W}_Q$  and the cosmological parameters  $\overline{W}_C = (\Omega_Q, \Omega_b h^2, h, n_s, \tau, A_s)$  which are the dark energy density, the baryon density, the Hubble parameter, the scalar spectral index, the optical depth and the overall amplitude of the scalar fluctuations respectively. We have modified a version of the CMBFAST code [21] to include the dark energy perturbation equations in terms

of the time derivatives of the equation of state [22]. In order to break the geometric degeneracy between  $w_Q^0$ ,  $\Omega_Q$  and  $h$ , we use the most recent compilation of supernova data of [4] in addition to the WMAP TT and TE power spectrum data. We evaluate the likelihood of CMB data with the help of the software provided by the WMAP team [23]. The important point which we want to stress is that we are able to treat both data sets, (WMAP and SN-Ia) without making any prior assumptions concerning the underlying cosmological model, in order to be as conservative as possible and to evade potential problems with issues like relative normalisations and bias. We restrict our analysis to dark energy models that satisfy the null dominant energy condition and  $w_Q^0, w_Q^m \geq -1$  and following the analysis by the WMAP team, we use the prior  $\tau \leq 0.3$  in order to prevent  $\Omega_b$  from taking unphysically high values.

### III. RESULTS

The WMAP CMB data constrains the cosmological parameters  $\overline{W}_C$  in a range of values consistent with the results of previous analysis such as [2, 24, 25]. In particular we find the scalar spectral index  $n_s = 1.00 \pm 0.04$ , the physical baryon density  $\Omega_b h^2 = 0.0234 \pm 0.0014$  and the optical depth  $\tau = 0.17 \pm 0.06$ . As mentioned above, in order to break the degeneracy between  $w_Q^0$ ,  $\Omega_Q$  and  $h$ , we combine the CMB data with the SN-Ia luminosity distance measurements. This allows us to constrain the Hubble constant to be  $h = 0.68 \pm 0.03$ , in agreement with the HST value [26], the dark energy density  $\Omega_Q = 0.72 \pm 0.04$  (all limits so far at  $1\sigma$ ) and the present value of the equation of state  $w_Q^0 < -0.82$  (at 95% CL). It is important to stress that the addition of the dark energy parameters  $\overline{W}_Q$  does not introduce any new degeneracies with the other parameters. This is clear from the fact that the constraints on  $\overline{W}_C$  are in agreement with other previous data analyses. Figure 1 shows the marginalised one-dimensional likelihoods for  $\Lambda$ CDM and the dynamic dark energy models. We will defer a detailed discussion of these results to a later paper, and in this paper concentrate on the use of dark matter clustering as a probe of quintessence models.

In general we expect dark energy to affect the value of  $\sigma_8$  because it can lead to a different expansion history of the universe [27]. However, in [15] it was shown that different dark energy models leave particular imprints on the large angular scales of the CMB anisotropy power spectrum through the integrated Sachs-Wolfe (ISW) effect. The excess of power produced by the ISW at low multipoles affects the normalization of the matter power spectrum [28]. For instance models with a fast late time transition in the equation of state produce a larger ISW effect than a pure cosmological constant scenario. As a consequence they require a smaller amplitude of primordial fluctuations in order to match the observed CMB spectrum. In this case the predicted value of  $\sigma_8$  will be

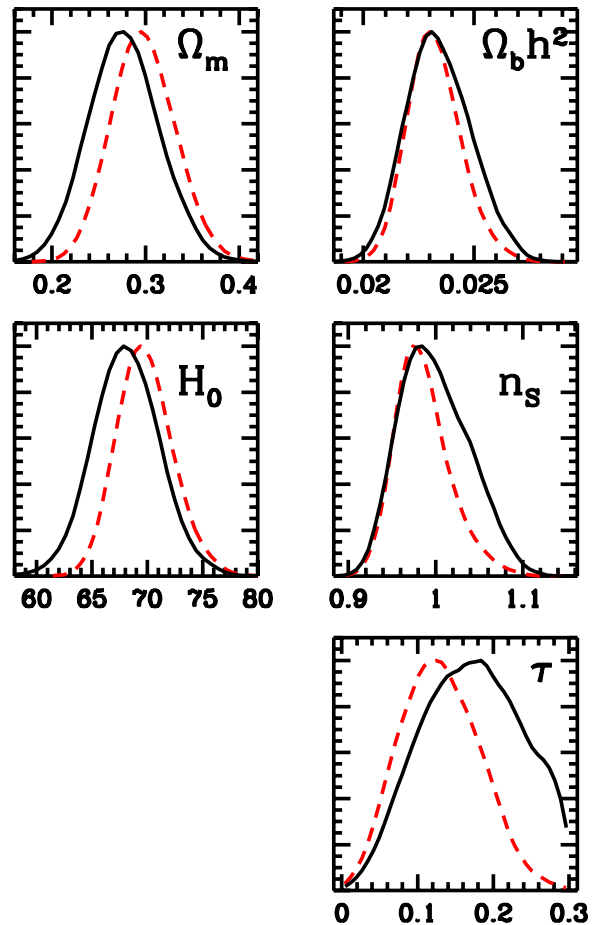


FIG. 1: Marginalized likelihoods for the various cosmological parameters in the  $\Lambda$ CDM scenario (red dashed curve) and including the QCDM models (black curve). The results agree very well with each other.

smaller than in the  $\Lambda$ CDM model. This specific class of models has already been investigated using pre-WMAP data [12, 29], but the results underestimated the optical depth subsequently found by WMAP, leading to an over-estimation of the power on small angular scales. It is only with the release of the first year of WMAP data that through one CMB data set, we can link the anisotropies on large and small angular scales. This is an exciting feature of the data, as it allows us to properly assess the effects of ISW and the normalization of the matter power spectrum. In figure 2 we plot the two dimensional likelihood contours in the  $\Omega_m - \sigma_8$  plane. The filled contours correspond to  $1$  and  $2\sigma$  values for the dark energy models spanned by  $\overline{W}_Q$ , while the solid curves correspond to the  $\Lambda$ CDM case. As expected from the above discussion, we note that  $\Lambda$  models have systematically higher values of  $\sigma_8$  than models with a time varying equation of state.

It seems clear that a CMB independent estimate of the value of  $\sigma_8$  would be able to distinguish between a  $\Lambda$ CDM

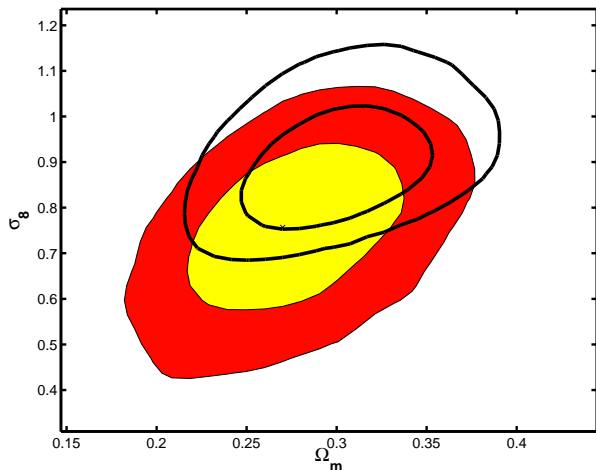


FIG. 2: Marginalized 68% and 95% confidence contours for quintessence (filled contours) and  $\Lambda$ CDM models (solid lines).  $\Lambda$ CDM has a systematically higher value of  $\sigma_8$ , and a slightly higher value of  $\Omega_m$ .

and dynamical equation of state model. For instance values of  $\sigma_8 < 0.7$  would be rejected at  $2\sigma$  in the  $\Lambda$ CDM case. More specifically we find that the value of  $\sigma_8$  can discriminate between different dark energy models. This can be seen in figures 3, 4 and 5 which are the main result of this paper. In Fig. 3 we plot the average value of  $\sigma_8$  as a function of  $a_c^m$  and  $w_Q^0$ , where the average is taken over all models in our chain which exhibit a rapid transition (defined here as  $w_Q^m > -0.2$  and  $\Delta < 0.1$ ). A  $\Lambda$ CDM model corresponds to  $a_c^m \rightarrow 0$  and  $w_Q^0 = -1$ . The average value of  $\sigma_8$  in this point is 0.9. As we move away from the  $\Lambda$ CDM corner, the average  $\sigma_8$  decreases monotonically, as seen by the contours. To assess the usefulness of  $\sigma_8$  for distinguishing between models given today's data, we also plot two 68% confidence regions, one for models with  $\sigma_8 > 0.9$  (lighter gray) and one with  $\sigma_8 < 0.6$  (darker gray). Clearly, if we restrict ourselves to models with a high value of  $\sigma_8$ , we favour a  $\Lambda$ CDM-like behaviour of the dark energy. In the opposite case, we find  $a_c^m \gtrsim 0.3$ . Together with the fast-transition conditions given above, this means that these models have an equation of state  $w(z > 2) \gg -1$ , and we would exclude the case  $p = -\rho$  at over 95% CL. As we marginalise over all other parameters, we see that no degeneracies spoil this result.

As a complementary view, we can plot  $a_c^m$  and  $w_Q^m$  for fast-transition models (without the condition on  $w_Q^m$ ), see Fig. 4. The data requires that  $w_Q^0 < -0.8$  and so  $\Lambda$ CDM models occupy the region defined by either  $a_c^m \rightarrow 0$  (in which case the equation of state is independent of  $w_Q^m$ ) or  $w_Q^m \rightarrow -1$  (and thus  $w(z) \approx -1$  without transition), which again coincides with the high- $\sigma_8$  models. Models with  $\sigma_8 < 0.6$  on the other hand require both  $a_c^m \gtrsim 0.3$  and  $w_Q^m \gtrsim -0.7$  at 68% CL.

Fig. 5 is the corresponding figure for dark energy models with a slowly varying equation of state ( $0 < a_c^m/\Delta < 0.8$ ). In this case the relevant parameters are  $w_Q^m$  and  $w_Q^0$ , and the  $\Lambda$ CDM models are now at  $w_Q^0 = w_Q^m = -1$ . Again,  $\sigma_8$  decreases rapidly as we move away from that corner. We show once more the  $1\sigma$  regions for models with  $\sigma_8 > 0.9$  (lighter gray) and with  $\sigma_8 < 0.6$  (darker gray). Models with a high value of  $\sigma_8$  are again clustered around the  $\Lambda$ CDM region, and those with a low clustering amplitude require  $w \gg -1$  at high redshift.

We expect these regions to shrink as the cosmological parameters become more constrained by future data, which will improve the impact of clustering as a probe of the time dependence of the dark energy. This is our main result, and it means that, given a precise measurement of  $\sigma_8$ , we can impose strong limits not only on the value of  $w$  today, *but also at earlier times*. Even if  $w_Q^0 \approx -1$  today, we are able to probe its behaviour at higher redshift and to either exclude  $\Lambda$ CDM or significantly constrain quintessence type models. Although especially slowly varying models cannot be ruled out as they can approximate the behaviour of a true cosmological constant arbitrarily closely, these models become less and less attractive as they start to require the same fine tuning as  $\Lambda$  itself.

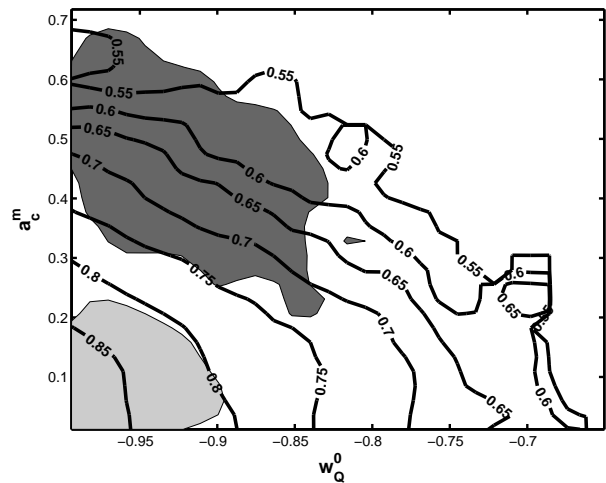


FIG. 3: The average  $\sigma_8$  as a function of  $w_Q^0$  and  $a_c^m$  for models with a rapid transition in  $w_Q$  (numbered lines). We also show the 68% confidence regions for models with  $\sigma_8 < 0.6$  (dark gray) and  $\sigma_8 > 0.9$  (light gray).

Why are we using  $\sigma_8$  as a variable as opposed to simply choosing one of the many published measured values of  $\sigma_8$ ? First, the published data shows a large spread of values [30], so that our conclusions would strongly depend on the choice of data sets. Secondly, the measurements also depend in general on the dark energy parameters and the results quoted are only valid for  $\Lambda$ CDM models. For example, this is the case for the large scale structure results, which implicitly assume a  $\Lambda$ CDM model when passing from redshift space to real space, and for weak lens-

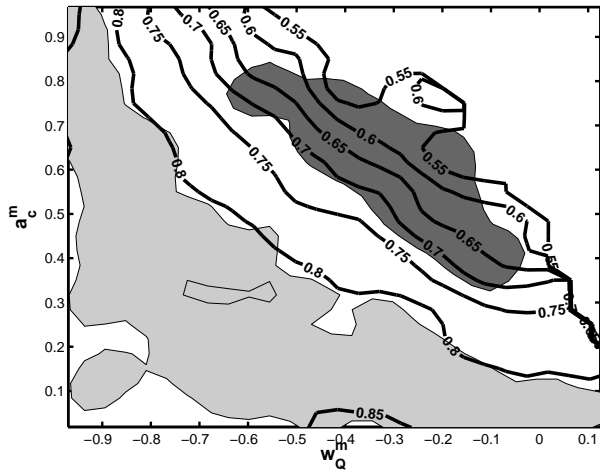


FIG. 4: The average  $\sigma_8$  as a function of  $w_Q^m$  and  $a_c^m$  for models with a rapid transition in  $w_Q$  (numbered lines). We also show the 68% confidence regions for models with  $\sigma_8 < 0.6$  (dark gray) and  $\sigma_8 > 0.9$  (light gray).

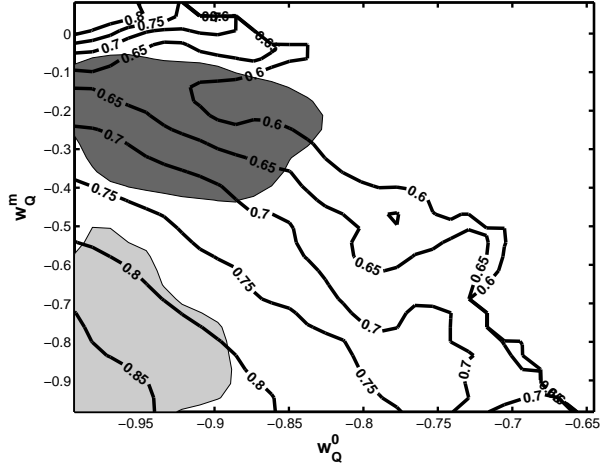


FIG. 5: The average  $\sigma_8$  as a function of  $w_Q^0$  and  $w_Q^m$  for models with a smoothly varying  $w_Q$  (numbered lines). We also show the 68% confidence regions for models with  $\sigma_8 < 0.6$  (dark gray) and  $\sigma_8 > 0.9$  (light gray).

ing measurements. In the second case, the dependence on the dark energy characteristics is strong enough that it can be used to constrain the evolution of the equation of state [31]. As an illustration, we can assume that the clustering results deduced from velocity fields in Ref. [32] are unaffected by the details of the dark energy evolution. As a rough approximation to their PSCz results, we set  $\sigma_8 \approx (1.13 \pm 0.05)(\Omega_m/0.3)^{0.6}$ . In this case, the constraints on quintessence models become much stronger, e.g.  $w_Q^0 < -0.9$  at 95% CL. On the other hand, if future precision measurements converge on  $\sigma_8 \lesssim 0.7$  then  $\Lambda$ CDM is ruled out at high significance.

Moreover,  $\sigma_8$  is linked to the amplitude of the mat-

ter power spectrum  $P(k)$  on small scales. To measure a possible running of the scalar spectral index,  $dn_S/d\log k$ , in inflationary models, it is necessary to combine CMB data on large scales with  $P(k)$  on small scales. Since quintessence models can change the amount of clustering on small scales with respect to a  $\Lambda$ CDM model, it is possible for them to mimic the effect of such a running. This possibility should be kept in mind when constraining models through the combination of different data sets [27].

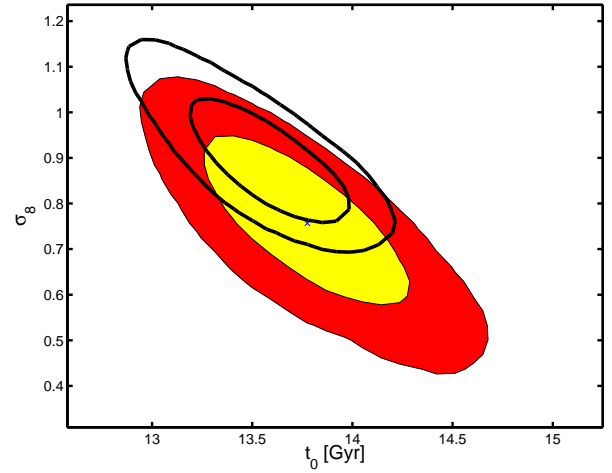


FIG. 6: Marginalized 68% and 95% confidence contours for quintessence (filled contours) and  $\Lambda$ CDM models (solid lines).

Another observable which has been studied in this context is the age of the universe,  $t_0$  [33, 34], which is in general also a function of the dark energy parameters  $\bar{W}_Q$ . An independent measurement of  $t_0$  (for which the WMAP limit does *not* qualify, as it explicitly assumes  $\Lambda$ CDM) can thus be used to set limits on the equation of state. Since the luminosity distance  $d_L$  and  $t_0$  possess a similar dependence on the Hubble rate, the SN-Ia data, which probe about two-thirds of the age of universe, can provide tight constraints on  $t_0$  even for generic dark energy models. For example in [4] considering  $\Lambda$ CDM cosmologies, the authors obtain  $H_0 t_0 = 0.96 \pm 0.04$ . The limit is also valid for quintessence, as we find  $H_0 t_0 = 0.96 \pm 0.03$  for the combination of CMB and SN-Ia data. This constraint, together with the remaining slight degeneracy in  $H_0$  which leads to lower values of the Hubble constant as we move away from the  $\Lambda$ CDM models, means that the allowed quintessence models are older than the those with a cosmological constant, as we can see in Fig. 6. The marginalised age of quintessence universes is  $t_0 = 13.8 \pm 0.3$  Gyr, while in the  $\Lambda$ CDM case  $t_0 = 13.55 \pm 0.26$ . Clearly, it will be difficult to use  $t_0$  to disentangle different models until the uncertainty in the cosmological parameters is further reduced. But if we were to find a lower limit on the age of the universe which is too high for  $\Lambda$ CDM, we could potentially interpret it to be a sign of quintessence.

## IV. CONCLUSIONS

In this paper we have demonstrated how, by combining WMAP and SN-Ia data, it is possible to use the normalisation of the dark energy power spectrum on cluster scales,  $\sigma_8$ , to discriminate between dynamical models of dark energy (quintessence models) and a conventional cosmological constant model ( $\Lambda$ CDM). In particular we have shown for the first time that a CMB independent measurement of  $\sigma_8$  allows us to constrain the parameters describing the evolution of the dark energy equation of state. For instance, we found that standard  $\Lambda$ CDM is ruled out at over 95% CL (compared to a time dependent dark energy component) if  $\sigma_8 < 0.7$ . This constraint can be relaxed by going beyond the standard model, i.e. introducing very massive neutrinos or a running of the spectral index [35]. However, we expect improved data

to lead to stronger limits in the near future. We have also briefly discussed the use of the age of the universe  $t_0$  as a way of constraining dark energy models, and shown that by itself it does not discriminate between quintessence and  $\Lambda$ CDM models, although coupled with  $\sigma_8$ , it may act as a useful cross check.

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